

Matt 109 Business Mathematics Notes

Spring 2013

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Chapter 3 Review: Decimals

Millions			Thousands			Units											
Hundred millions (100,000,000)	Ten millions (10,000,000)	Millions (1,000,000)	Hundred thousands (100,000)	Ten thousands (10,000)	Thousands (1,000)	Hundreds (100)	Tens (10)	Ones (1)	Decimal point	Tenths 0.1	Hundredths 0.01	Thousandths 0.001	Ten-thousandths 0.0001	Hundred-thousandths 0.00001	Millionths 0.000001	Ten-millionths 0.0000001	Hundred-millionths 0.00000001
					2	3	1	5	.	6	2	7	4	3	2		

Example 1: Write out the number: 1028.45

Example 2: Round 17.3754 as instructed:

- A) to the nearest hundredth
- B) to the nearest tenth

Example 3: Add: $32 + 2.55 + 8.85 + 0.625$

Example 4: Subtract: $26.30 - 15.384$

Example 5: Multiply: 2.35×0.015

Example 6: Multiply 36.54×100

Example 7: Divide 5.59 by 17

Example 8: Divide 23.71 by 10

Chapter 5 Review: Equations

Example 1: Solve: $2x = 18$

Example 2: Solve: $\frac{A}{4} = 5$

Example 3: Solve: $N + 17 = 24$

Example 4: Solve: $Y - 7 = 12$

Example 5: Solve: $3N - 1 = 14$

Example 6: Solve: $\frac{X}{5} - 3 = 1$

Example 7: Solve: $A + 3A - 2 = 14$

Example 8: Solve: $2(3Y + 1) = 14$

Example 9: Solve: $\frac{3}{8} = \frac{21}{N}$

Example 10: If full-time employees work 4 hours longer than part-time employees and part-time employees work 6 hours, how long do full-time employees work for?

Example 11: Wanda plans to save one tenth of her salary. If her salary is \$35,000, how much will she save?

Example 12: In a group of 600 people, there are twice as many men as there are women. How many men and women are there?

Example 13: Your car gets 23 miles to the gallon of gas. How far can you go on 16 gallons of gas?

Example 14: Given the formula $S = C + M$, where S is the selling price, C is the cost, and M is the markup, find the selling price of a television that costs \$875 marked up by \$400.

Example 15: Solve the formula $U = \frac{P}{N}$ for P .

Chapter 7 Review: Percents

Example 1: Rewrite the decimal as a percent:

- A) 0.27
- B) 0.875
- C) 1.73
- D) 0.004
- E) 2

Example 2: Write the percent as a fraction:

- A) 37 %
- B) 26.5 %
- C) 127 %
- D) 7 %
- E) 0.9 %

Percentage Formula: $P = RB$, where B is the base (original number or quantity), P is portion (a part of the base), and R is the rate (percent).

Example 3: 20 % of 75 is what number?

Example 4: What percent of 50 is 30?

Example 5: Eight is 10 % of what number?

Example 6: If $66\frac{2}{3}$ % of 900 employees are on the preferred insurance plan, how many people are on the plan?

Example 7: If 20 cars were sold from a lot that had 50 cars, what percent of the cars were sold?

Amount Increase: Amount of Increase = New Amount – Beginning Amount

Decrease (is new amount is smaller than beginning amount):

Amount of Decrease = Beginning Amount – New Amount

Example 8: David's salary increased from \$58,240 to \$63,190. What is the amount of increase?

Example 9: A coat was marked down from \$98 to \$79. What is the amount of markdown?

Amount of Change: Amount of Change = Percent of Change x Original Amount

Example 10: You will receive a 3.2 % raise. If your salary is \$42,560, how much will your raise be?

Section 7.2: Measures of Central Tendency

Mean: $\text{mean} = \frac{\text{sum of values}}{\text{number of values}}$

Example 1: Find the mean: 780, 620, 198, 457, 780, 215, and 41.

Median: the value in the middle when the numbers are arranged from smallest to largest or largest to smallest.

Example 2: Find the median: 780, 620, 198, 457, 780, 215, and 41.

Mode: The value that occurs the most frequently. If no value occurs the most frequently, there is no mode.

Example 3: Find the mode: 780, 620, 198, 457, 780, 215, and 41.

Grouped Frequency Distribution: A list that groups together data into class intervals and tallies how many pieces of data belong to each class interval.

Example 4: Use the grouped frequency distribution to answer the questions:

TABLE 7-11 Frequency Distribution of 25 Scores		
Class Interval	Tally	Class Frequency
60–64	/	1
65–69	/	1
70–74	//	2
75–79	/	6
80–84		3
85–89		5
90–94		5
95–99	//	2
		<u>25</u>

- How many students made As (90s)?
- What percent of the total grades were As?
- What percent of students passed (70 or higher)?

Example 5: The following grades were earned on a math test:

78 79 84 61 67 69 91 87 72 76 63 69 95 87 94

Make a frequency distribution of the data using the intervals 60-69, 70-79, 80-89, and 90-99.

Section 8.1: Single Trade Discounts

Trade Discount: $T = RL$, where T is the trade discount, R is the single trade discount rate and L is the list price.

Example 1: The list price of a refrigerator is \$1,200. A store can buy the refrigerator at the list price less 20 %. Find the trade discount and the net price of the refrigerator.

Complement of a Percent: the difference between 100 % and the given percent, i.e., the complement of 35 % is 65%.

Example 2: What is the compliment of 17%?

Example 3: A store orders 300 pens that list for \$0.30 each, 200 pads that list for \$0.60 each, and 100 boxes of paper clips that list for \$0.90 each. The single trade discount rate for the order is 12%. Find the net price of the order.

Section 8.2: Trade Discount Series

Trade Discount Series: Multiple discounts offer one after the other, written like x/y/z. For example, 20/15/10 means 20 % taken off, then 5 %, then 10 %.

Example 1: What is the net price of an item that lists for \$600 with a trade discount of 15/10/5?

Example 2: Store A sells an item for \$700 with a discount series of 20/10/10. Store B sells the same item for \$560 with a discount series of 10/10/10. Which store has the better deal?

Single Discount Equivalent: A discount series lumped as a single discount.

Total amount of a series of discounts = Single Discount Equivalent x List Price

Net amount you pay after a series of discounts = Net Discount Equivalent x List Price

Example 3: An item costing \$1,500 has a discount series of 30/20/10. What is the single discount equivalent?

Section 8.3: Cash Discounts and Sale Terms

Cash Discount: $\text{Cash Discount} = \text{Cash Discount Rate} \times \text{Net Price}$

Usually include a date. Example: 2/10, n/30 means 2 % will be discounted if the bill is paid within 10 days of the invoice date and the full (or net) amount is due within 30 days.

Example 1: A \$450 item comes on July 27 with terms 2/10, n/30. Find the latest date the cash discount is allowed and the cash discount.

Net Amount: $\text{Net Amount} = \text{Net Price} - \text{Cash Discount}$

$\text{Net Amount} = \text{Compliment of Cash Discount Rate} \times \text{Net Price}$

Example 2: Find the net amount for the previous example.

End-of-Month (EOM) Terms: Discounts allowed is the bill is paid during the first n days of the next month. For example, 2/10 EOM means 2% discount if the bill is paid during the first 10 days of the month *after* the month of the invoice. So if the bill is dated November 19, a 2% discount is allowed if the bill is paid on or before December 10.

If the invoice is dated on or after the 26th of the month, the discount is allowed if the bill is paid during the first n days of the month after the *next* month. So if our previous example had an invoice date November 26, the discount is allowed until January 10.

Example 3: If you receive a \$200 bill dated April 27 with 3/10 EOM, how much will you pay if you get the terms and when must you pay by?

Receipt-of-Goods (ROG) Terms: The date the goods are received. 1/10 ROG means a 1% discount is allowed if the bill is paid within 10 days of receipt of goods.

Example 4: An invoice is dated November 9 for \$400 and has sales terms 2/10 ROG. The items arrive November 13. If the bill is paid on November 21, what is the net amount due? If the bill is paid on December 2, what is the net amount due?

Partial Payments: Made to take advantage of cash discounts but don't cover the entire price.

$$\text{Amount credited} = \frac{\text{partial payment}}{\text{complment of cash discount rate}}$$

$$\text{Outstanding Balance} = \text{Net Price} - \text{Amount Credited}$$

Example 5: A company receives an invoice for \$875 with terms 3/10, n/30. The company could not pay the entire bill within 10 days but sent a check for \$500. What amount was credited to the company?

Free on Board (FOB): List of who pays shipping and when. Cash discounts do not apply to shipping.

FOB shipping point: FOB at shipping point.

Freight collect: buyer pays shipping when shipment is received.

FOB destination: FOB at destination.

Freight paid: seller pays shipping when items are shipped.

Prepay and add: seller pays shipping when items are shipped, but shipping costs are added to invoice for the buyer to pay.

Example 6: Calculate the cash discount and the net amount paid for a \$800 order with sales terms of 3/10, 1/15, n/30 if the cost of shipping was \$40 (which is included in the \$800). The invoice was dated June 13, marked freight prepay and add, and paid June 24.

Section 9.1: Markup Based on Cost

$S = C + M$, or Selling Price = Cost + Markup

Example 1: What is the selling price if the cost is \$28.35 and the markup is \$5.64?

Example 2: A store buys an item for \$2.45 and sells it for \$5.88. What is the markup?

Example 3: A store sells an item for \$1.29. If the markup is \$0.35, what is the cost?

Markup Based on Percent of Cost: Markup = Rate of Markup $\times$ Cost

Example 4: A store buys an item for \$9. If the item is sold for \$15, what is the percent markup based on cost? Round to the nearest tenth of a percent.

Example 5: A store pays \$5 for an item and sells them at a 50% markup based on cost. Find the selling price.

Example 6: An item sells for \$20. The markup rate is 50% of the cost. Find the cost of the item and the markup.

Section 9.2: Markup Based on Selling Price and Markup Comparisons

Markup = Rate of Markup x Selling Price

Example 1: An item costs \$4 and sells for \$10. Find the rate of markup based on the selling price.

Example 2: Find the cost and selling price if an item is marked up \$5 with a 20% markup rate based on selling price.

Example 3: Find the selling price and the markup for an item that costs \$28 and is marked up 30% of the selling price.

Example 4: Find the markup and cost of an item that sells for \$2.99 and is marked up 25% of the selling price.

Example 5: Find the rate of markup based on cost and based on selling price of an item that costs \$1,500 and sells for \$2,000.

Convert Markup Based on Selling to Markup Based on Cost: $M\%_{\text{cost}} = \frac{M\%_{\text{selling}}}{100\% - M\%_{\text{selling}}} (100\%)$

$$M\%_{\text{selling}} = \frac{M\%_{\text{cost}}}{100\% - M\%_{\text{cost}}} (100\%)$$

Example 6: An item is marked up 30% based on selling price. What is the equivalent markup based on the cost?

Example 7: An item is marked up 40% based on cost. What is the markup rate based on selling price?

Section 9.3: Markdown, Series of Markdowns, and Perishables

Markdown: Amount original price is decreased by.

Markdown = Original Selling Price – Reduced Price, or $M = S - N$

$$\text{Rate of Markdown} = \frac{\text{amount of markdown}}{\text{original selling price}} \text{ or } M\% = \frac{M}{S} (100\%)$$

Example 1: An item originally sold for \$36 and was marked down to sell for \$30. Find the markdown and the rate of markdown.

Example 2: An item was originally priced at \$12 and was reduced by 25%. Find the markdown and the sale (new) price.

Example 3: An item costing \$145 is on a 40% off sale. If a customer has a coupon that reads “take an additional 10% off any already reduced price,” how much will the customer pay for the item?

Section 11.1: Simple Interest Formula

Simple Interest: $I = PRT$ or Interest = Principal x Rate x Time

Example 1: Find the interest paid on a loan of \$1,500 for one year at a rate of 9%.

Example 2: Find the interest paid on a loan of \$5,000 at 8.5% for 2 years.

Maturity Value: $MV = P + I$ or Maturity Value = Principal + Interest

Example 3: Find the maturity value for example 2.

Example 4: Find the interest paid and the maturity value of a loan of \$2,500 at 3.5% for 45 months.

Example 5: If I paid \$202.50 interest on a \$1,800 loan over 1.5 years, what was the interest rate?

Section 11.2: Ordinary and Exact Interest

Exact Time: exact number of days in a time period.

Example 1: Find the exact time from January 12 to September 18 in a normal year and on a leap year.

Due Date: When the number of days for a loan is set before looking at a calendar.

Example 2: Find the due-date for a 90-day loan made on October 7.

Ordinary Interest: Use 360 as the number of days in a year.

Exact Interest: Use 365 as the number of days in a year.

Example 3: Find the ordinary interest for a loan of \$500 at 7%. The loan was from March 15 to May 15.

Example 4: Find the exact interest for a loan of \$500 at 7%. The loan was from March 15 to May 15.

Section 11.3: Promissory Notes

Bank Discount and Proceeds: Use ordinary interest.

Back discount = Face Value x Discount Rate x Time, or $I = PRT$

Proceeds = Face Value – Bank Discount, or $A = P - I$

Example 1: Find the bank discount and the proceeds on a promissory note for \$4,000 at 8% from June 5 to September 5.

True or Effective Interest Rate:

Bank Discount: $I = PRT$

Proceeds: Proceeds = Principle – Bank Discount

Effective Interest Rate: $R = \frac{I}{PT}$ using proceeds as the principle.

Example 2: What is the effective interest rate of a simple discount note for \$5,000, at an ordinary bank discount rate of 12%, for 90 days?

Section 13.1: Compound Interest and Future Value

Period of Interest Rate: $\text{Period of Interest Rate} = \frac{\text{Annual Interest Rate}}{\text{Number of Interest Periods Per Year}}$

Future Value: Total amount of money. P is principal, r is the interest rate, t is the time in years, and n is the number of times compounded (annual = 1, semiannual = 2, quarterly = 4, monthly = 12).

$$FV = P\left(1 + \frac{r}{n}\right)^{nt}$$

Compound Interest: $I = FV - P$

Example 1: If I take out a loan for \$8,000 for three years at 9% compounded annually, find the future value and the compound interest.

Example 2: Find the future value of a \$10,000 investment at 2.3% compounded semiannually for 5 years.

Example 3: I have \$10,000 to invest for 2 years. Plan A is at 8% compounded quarterly. Plan B is at 8.2% compounded annually. Which plan is better?

13.2: Present Value

Present Value: Amount to be invested to get a certain amount in the future.

$$\text{For one year: } PV = \frac{FV}{1+r}$$

$$\text{For } t \text{ years: } PV = \frac{FV}{\left(1 + \frac{r}{n}\right)^{nt}}$$

Example 1: Find the amount of money that needs to be set aside now to ensure that \$10,000 will be available in one year at 4% compounded annually.

Example 2: Find the amount of money that needs to be set aside now to ensure that \$8,000 will be available in three years at 5.2% compounded monthly.

Section 14.1: Future Value of an Annuity

Future Value: how much an investment is worth is periodic payments are made over the time of the investment. PMT is amount of annuity payment.

Ordinary Annuity: payments are made at the end of each period.

$$FV_{\text{ordinary annuity}} = PMT \left(\frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}} \right)$$

Annuity Due: payments are made at the beginning of each period.

$$FV_{\text{annuity due}} = PMT \left(\frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}} \right) \left(1 + \frac{r}{n}\right)$$

Example 1: Find the future value of an ordinary annuity of \$100 paid monthly at 5.25% for 10 years.

Example 2: Find the future value of an annuity due of \$50 monthly at 5.75% for 5 years.

Example 3: I have \$150 to invest monthly for 7 years. Plan A is an ordinary annuity at 5.2%. Plan B is annuity due at 5%. Which plan is better?

Section 14.2: Sinking Funds and the Present Value of an Annuity

Sinking Fund: Payment into an ordinary annuity when the Future Value is known but the annuity payment is unknown. PV is the periodic payment that is made into the fund.

$$PMT_{ordinary \text{ annuity}} = FV \left(\frac{\frac{r}{n}}{\left(1 + \frac{r}{n}\right)^{nt} - 1} \right)$$

$$PV_{ordinary \text{ annuity}} = PMT \left(\frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n} \left(1 + \frac{r}{n}\right)^{nt}} \right)$$

Example 1: I want \$100,000 for a retirement fund in 20 years. With 5.5% annual interest, how much do I have to contribute each month to reach my goal?

Example 2: Upon retiring, I will draw monthly payments from a fund. How much do I need in a fund that pays 5.5% interest to receive \$700 per month payment for 20 years?

Section 16.1: Mortgage Payments

TABLE 16-1

Monthly Payment of Principal and Interest per \$1,000 of Amount Financed

Years financed	Annual interest rate															
	4.00%	4.25%	4.50%	4.75%	5.00%	5.25%	5.50%	5.75%	6.00%	6.25%	6.50%	6.75%	7.00%	7.25%	7.50%	7.75%
10	10.12	10.24	10.36	10.48	10.61	10.73	10.85	10.98	11.10	11.23	11.35	11.48	11.61	11.74	11.87	12.00
12	8.76	8.88	9.00	9.12	9.25	9.37	9.50	9.63	9.76	9.89	10.02	10.15	10.28	10.42	10.55	10.69
15	7.40	7.52	7.65	7.78	7.91	8.04	8.17	8.30	8.44	8.57	8.71	8.85	8.99	9.13	9.27	9.41
17	6.76	6.89	7.02	7.15	7.29	7.42	7.56	7.69	7.83	7.97	8.11	8.25	8.40	8.54	8.69	8.83
20	6.06	6.19	6.33	6.46	6.60	6.74	6.88	7.02	7.16	7.31	7.46	7.60	7.75	7.90	8.06	8.21
22	5.70	5.84	5.97	6.11	6.25	6.39	6.54	6.68	6.83	6.98	7.13	7.28	7.43	7.59	7.75	7.90
25	5.28	5.42	5.56	5.70	5.85	5.99	6.14	6.29	6.44	6.60	6.75	6.91	7.07	7.23	7.39	7.55
30	4.77	4.92	5.07	5.22	5.37	5.52	5.68	5.84	6.00	6.16	6.32	6.49	6.65	6.82	6.99	7.16
35	4.43	4.58	4.73	4.89	5.05	5.21	5.37	5.54	5.70	5.87	6.04	6.21	6.39	6.56	6.74	6.92

Monthly Mortgage Payment using chart: $\frac{\text{amount financed}}{\$1,000} \times \text{table value}$

Example 1: Using the above chart, find the monthly mortgage payment for a \$212,000 home on a 30-year fixed-rate loan at 6% annual interest if a 20% down payment is made.

Monthly Mortgage Payment:
$$M = P \left(\frac{\frac{r}{n}}{1 - \left(1 + \frac{r}{n}\right)^{-n}} \right)$$

Example 2: Find the monthly mortgage payment for a home costing \$179,500 at 6% annually for 30 years if a 15% down payment is made.

Total Interest on a Mortgage:

$$\text{Total Interest} = \text{Number of Payments} \times \text{Amount of Payment} - \text{Amount Financed}$$

Example 2: Calculated the total interest paid on the fixed-rate loan of \$159,600 for 30 years at 6% interest rate using the monthly payment found in example 2.

Section 16.2: Amortization Schedules and Qualifying Ratios

Amortization Schedule: shows the amount of principal and interest for each payment of the loan.

For First Month:

Interest Portion of First Monthly Payment = Original Principal x Monthly Interest Rate

Principal Portion of First Monthly Payment =

$$\text{Monthly Payment} - \text{Interest Portion of First Monthly Payment}$$

First End-of-Month Principal = Original Principal – Principal Portion of First Monthly Payment

For Each Remaining Month:

Interest Portion of Monthly Payment = Previous End-of-Month Principal x Monthly Interest Rate

Principal Portion of Monthly Payment = Monthly Payment – Interest Portion of Monthly Payment

End-of-Month Principal = Previous End-of-Month Principal – Principal Portion of Monthly Payment

Example 1: Complete first 2 rows of the amortization schedule for a \$69,900 mortgage at 7% annual interest for 30 years. The monthly payments are \$462.84.

Qualifying Ratios: Used to determine lender's ability to repay the loan.

$$\text{Loan-to-Value Ratio (LTV): } LTV = \frac{\text{amount mortgaged}}{\text{appraised value of property}}$$

$$\text{Housing Ratio or Front-End Ratio: } \frac{\text{total mortgage payment (PITI)}}{\text{gross monthly income}}$$

$$\text{Debt-to-Income Ratio (DTI): } DTI = \frac{\text{total fixed monthly expenses}}{\text{gross monthly income}}$$

Example 2: Find the LTV for a home appraised at \$250,000 that the buyer will purchase for \$248,000. The down payment will be \$68,000.

Section 15.1: Stocks

Example 1: Use the stock listing provided.

- A) How many shares of AFLAC, or AFL, were traded?
- B) What is the difference between the high price and the low price of the day?
- C) What was the closing price the previous day?

$$\text{Current Yield: } \frac{\text{annual dividend per share}}{\text{closing price per share}} \times 100\%$$

Example 2: Find the current yield of AT&T stock that reported a dividend of \$1.68 and a closing price of \$26.28.

Trailing Earnings: a company's earnings-per-share for the past 12 months.

$$\text{P/E ratio} = \frac{\text{current price per share}}{\text{net income per share (past 12 months)}}$$

Example 3: Find the P/E ratio of a corporation that reported last year's net income at \$6.16 per share if the stock sells for \$58 per share.