

Lecture Notes

Math 013 – Algebra I

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Section 2.3: Solving Linear Equations

Like Terms: terms with exactly the same variables raised to exactly the same powers.

Example 1: Combine like terms:

A) $7x - 3x$

B) $10y^2 + y^2$

Example 2: Simplify each expression:

A) $2x + 3x + 5 + 2$

B) $-5a - 3 + a + 2$

C) $3(2x - 5) + 1$

D) $-2(4x + 7) - (3x - 1)$

E) $9 - 3(4x + 10)$

Linear Equation in 1 Variable: of the form $ax + b = c$

Addition Property of Equality: $a = b$ and $a + c = b + c$ are equivalent.

Example 3: Solve:

$$x - 7 = 10$$

Example 4: Solve:

$$2x + 3x - 5 + 7 = 10x + 3 - 6x - 4$$

Example 5: Solve:

$$8x - 5x - 3 + 9 = x + x + 3 - 7$$

Example 6: Solve:

$$7 = -5(2a - 1) - (-11a + 6)$$

Example 7: Solve:

$$-3 = 3(2y - 5) - (5y + 1)$$

Multiplication Property of Equality: If $c \neq 0$, then $a = b$ and $ac = bc$ are equivalent

Example 8: Solve:

$$5x = 15$$

Example 9: Solve:

$$-3x = 33$$

Example 10: Solve:

$$\frac{y}{7} = 20$$

Example 11: Solve:

$$12a - 8a = 10 + 2a - 13 - 7$$

Example 12: Solve:

$$6b - 11b = 18 + 2b - 6 + 9$$

Solving Linear Equations:

Step 1: If there are fractions, multiply both sides by the lcd.

Step 2: Use the distributive property to get rid of parentheses.

Step 3: Combine like terms on each side.

Step 4: Get all the terms with the variable on one side of the equation and all the terms without the variable on the other side of the equation using the addition property of equality.

Step 5: Get the variable by itself with the multiplication property of equality.

Step 6: Check your answer by plugging it back in the original problem.

Example 13: Solve:

$$4(2x - 3) + 7 = 3x + 5$$

Example 14: Solve:

$$8(2 - t) = -5t$$

Example 15: Solve:

$$\frac{x}{2} - 1 = \frac{2}{3}x - 3$$

Example 16: Solve:

$$\frac{3}{5}x - 2 = \frac{2}{3}x - 1$$

Example 17: Solve:

$$\frac{2(a+3)}{3} = 6a + 2$$

Example 18: Solve:

$$\frac{3(x-2)}{5} = 3x + 6$$

Example 19: Solve:

$$0.25x + 0.10(x - 3) = 0.05(22)$$

Example 20: Solve:

$$-2(x - 5) + 10 = -3(x + 2) + x$$

If your variables disappear and you get a false statement, the answer is no solution.

Example 21: Solve:

$$3(x - 4) = 3x - 12$$

If your variables disappear and you get a true statement, the answer is all real numbers.

Section 2.6: Percent and Mixture Problems

Part = Percent · Total

Example 1: The number 63 is what percent of 72?

Example 2: The number 120 is 15% of what number?

Example 3: If an item that costs \$140 is reduced by 20%, how much does it cost?

Example 4: If an item that costs \$480 is reduced by 85%, how much does it cost?

Percent Increase/Decrease:

Increase = New Cost – Old Cost

Decrease = Old Cost – New Cost

Increase = Percent Increase · Old Cost

Decrease = Percent Decrease · Old Cost

Example 5: If the price went up from \$9258 to \$12,796, find the percent increase.

Example 6: If Bob got an 8% raise in his salary from last year and is making \$35,000 this year, how much did he make last year?

Example 7: How much 70% acid should be mixed with 10 liters of a 20% solution to get a 40 % solution?

Example 8: How much pure acid should be mixed with 5 gallons of a 25% solution to get a 50% solution?

Example 9: If 12 liters of a 50% acid solution are needed, how much of a 40% and a 70% solution should be mixed together?

Example 10: If 6 liters of a 3% solution is needed, how much of a 2% and a 5% should be mixed together?

Section 2.7: Further Problem Solving

Example 1: A bicyclist rode at an average speed of 18 mph on level roads and then slowed down to an average speed of 10 mph on hilly roads. If the bicyclist rode 98 miles total, how long did the entire trip take if it took the same amount of time to ride the level roads as it did the hilly roads?

Example 2: 2 trains leave at the same time. One travels north and the other travels south at a speed that is 15 mph faster. In 2 hours, the trains are 230 miles apart. Find the speed of each train.

Example 3: \$2420 is made up of \$10 bills and \$20 bills. If there are 37 more \$20 bills than \$10 bills, find how many of each denomination there are.

Section 2.8: Solving Linear Inequalities

Inequalities:

$<$	“less than”
$>$	“greater than”
$\leq$	“less than or equal to”
$\geq$	“greater than or equal to”
$\neq$	“not equal to”

For the most part, we solve inequalities the same way as equalities. There are two exceptions:

The inequality sign flips when:

1. Multiplying or dividing by a negative

$$\begin{array}{rcl} -2x & < & 4 \\ x & > & -2 \end{array}$$

2. Rewriting the inequality

$$\begin{array}{rcl} x & > & 3 \\ 3 & < & x \end{array}$$

When solving inequalities, we can graph our answer on a number line. We want our final answer to be in interval notation.

Interval Notation:

Write in the form (Smaller number, larger number) or [smaller number, larger number]

Use () for $<$ and $>$ Don't include the number

Use [] for $\leq$ and $\geq$ Do include the number

Use () for $\pm \infty$ (plus or minus infinity)

Example 1: Graph on a number line and write in interval notation:

$$x > 4$$

We use a parenthesis at 4 because we don't include 4 itself.

Since the arrow is shaded in, the largest number on the interval is ∞

So, our answer in interval notation is: $(4, \infty)$

Example 2: Graph on a number line and write in interval notation:

$$x \leq -1$$

Example 3: Graph on a number line and write in interval notation:

$$-7 < x \leq 10$$

Example 4: Solve and write your answer in interval notation:

$$x + 4 \leq -6$$

Example 5: Solve and write your answer in interval notation:

$$-2x < -4$$

Example 6: Solve and write your answer in interval notation:

$$-4x + 7 \geq -9$$

Example 7: Solve and write your answer in interval notation:

$$2x + 7 \leq x - 11$$

Example 8: Solve and write your answer in interval notation:

$$2(x - 3) - 5 \leq 3(x + 2) - 18$$

Example 9: Solve and write your answer in interval notation:

$$-1 \leq 2x - 3 \leq 5$$

Example 10: Solve and write your answer in interval notation:

$$3 \leq \frac{3x}{2} + 4 \leq 5$$

Example 11: Solve and write your answer in interval notation:

$$6 > \frac{3}{4}x + 5 > 1$$

Section 5.1: Exponents

Example 1: Evaluate:

A) 2^3

B) 3^1

C) -4^2

D) $(-4)^2$

Example 2: Evaluate $2x^3$ if $x = 5$.

Example 3: Evaluate $\frac{9}{x^2}$ if $x = -3$.

Product Rule for Exponents: $a^m \cdot a^n = a^{m+n}$

Example 4: Use the product rule to simplify:

A) $4^2 \cdot 4^3$

B) $x^4 \cdot x^6$

C) $y^3 \cdot y$

D) $z^3 \cdot z^2 \cdot z^7$

E) $a^2 \cdot b^2$

Example 5: Simplify:

A) $(2x^2)(-3x^5)$

B) $(x^2y)(x^3y^2)$

C) $(-a^7b^4)(3ab^9)$

Power Rule for Exponents: $(a^m)^n = a^{mn}$ Example 6: Use the power rule to simplify:

A) $(x^2)^5$

B) $(y^8)^2$

C) $[(-5)^3]^7$

Power of a Product Rule: $(ab)^n = a^n b^n$ Example 7: Simplify:

A) $(xy)^4$

B) $(2a)^3$

C) $\left(\frac{1}{3}mn^3\right)^2$

D) $(-5x^2y^3z)^2$

Power of a Quotient Rule: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ Example 8: Simplify:

A) $\left(\frac{x}{y}\right)^7$

B) $\left(\frac{x^3}{3y^5}\right)^4$

Quotient Rule for Exponents: $\frac{a^m}{a^n} = a^{m-n}$

Example 9: Simplify:

A) $\frac{x^5}{x^2}$

B) $\frac{4^7}{4^3}$

C) $\frac{(-3)^5}{(-3)^2}$

D) $\frac{x^2}{y^3}$

E) $\frac{2x^5y^2}{xy}$

F) $\frac{6a^3b^7}{ab^5}$

Zero Exponent: $a^0 = 1$, where $a \neq 0$.

Example 10: Simplify: 3^0

Example 11: Simplify:

A) $\left(\frac{st}{2}\right)^4$

B) $(9y^5z^7)^2$

C) $\left(\frac{-5x^2}{y^3}\right)^2$

Section 5.2: Adding and Subtracting Polynomials

Polynomial: a finite sum of terms of the form ax^n , where a is a real number and n is a whole number.

Monomial: A polynomial with one term.

Binomial: A polynomial with two terms.

Trinomial: A polynomial with three terms.

Degree of a Term: the sum of the exponents on the variables.

Example 1: Find the degree of each term:

A) $3x^2$

B) -2^3y^5

C) y

D) $12x^2yz^3$

E) 5

Degree of a polynomial: the greatest degree of any term.

Example 2: Find the degree of the polynomial:

A) $-2x^2 + 3x + 6$

B) $15x - 10$

C) $7x + 3x^3 + 2x^2 - 1$

Example 3: If $P(x) = 3x^2 - 2x - 5$, find the following:

A) $P(1)$
Let $x = 1$ and evaluate.

B) $P(-2)$

Example 4: Simplify each polynomial:

A) $-3x + 7x$

B) $x + 3x^2$

C) $11x^2 + 5 + 2x^2 - 7$

D) $7a^2 - 5 - 3a^2 - 7$

E) $\frac{2}{5}x^4 + \frac{2}{3}x^3 - x^2 + \frac{1}{10}x^4 - \frac{1}{6}x^3$

F) $-9x^2 + 3xy - 5y^2 + 7yx$

Adding Polynomials: To add, combine like terms.

Example 5: Add:

A) $(7x^3y - xy^3 + 11) + (6x^3y - 4)$

B) $(3a^3 - b + 2a - 5) + (a + b + 5)$

C) $(11x^3 - 12x^2 + x - 3) + (x^3 - 10x + 5)$

Subtracting Polynomials: To subtract, distribute the negative sign, then add.

Example 6: Subtract:

A) $(2x^3 + 8x^2 - 6x) - (2x^3 - x^2 + 1)$

B) $(3x^3 - 5x^2 + 4x) - (x^3 - x^2 + 6)$

C) $(9a^2b^2 + 6ab - 3ab^2) - (5b^2a + 2ab - 3 - 9b^2)$

Section 5.3: Multiplying Polynomials

Example 1: Multiply:

A) $6x \cdot 4x$

B) $-7x^2 \cdot 2x^5$

C) $5x(2x^3 + 6)$
Distribute.

D) $-3x^2(5x^2 + 6x - 1)$

Multiplying Polynomials: Multiply each term of the first polynomial by each term of the second polynomial then combine like terms.

Example 2: Multiply polynomials:

A) $(3x + 2)(2x - 5)$

B) $(5x - 2)(2x + 3)$

C) $(2x - y)^2$

D) $(5x - 3y)^2$

E) $(x + 2)(3x^2 - 4x + 2)$

F) $(y + 4)(2y^2 - 3y + 5)$

G) $(a + b)^3$

H) $(2x^2 - 3x + 4)(x^2 + 5x - 2)$

Section 5.4: Special Products

FOIL Method: To multiply 2 binomials, multiply the First terms, then the Outer terms, then the Inner terms, then the Last terms. Then add up like terms.

Example 1: Multiply with FOIL:

A) $(x - 3)(x + 4)$

B) $(x + 2)(x - 5)$

C) $(5x - 7)(x - 2)$

D) $(4x - 9)(x - 1)$

E) $2(y + 6)(2y - 1)$

F) $(3y + 1)^2$

Squaring a Binomial:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Example 2: Square the binomial:

A) $(x + 2)^2$

B) $(p - q)^2$

C) $(2x + 5)^2$

Multiplying the Sum and Difference of 2 Terms: $(a + b)(a - b) = a^2 - b^2$

Example 3: Multiply the sum and difference of 2 terms:

A) $(x + 4)(x - 4)$

B) $(6y + 7)(6y - 7)$

C) $(2p - 3q)(2p + 3q)$

Example 4: Multiply:

A) $3(x + 5)(x - 5)$

B) $(x - 7)(3x + 4)$

C) $(7x + 4)^2$

D) $(y^4 + 2)(3y^2 - 1)$

E) $8(9x - 1)^2$

Section 5.5: Negative Exponents and Scientific Notation

Negative Exponent: $a^{-n} = \frac{1}{a^n}$

Example 1: Simplify so that there are positive exponents only:

A) 3^{-2}

B) $2x^{-3}$

C) $2^{-1} + 4^{-1}$

D) $(-2)^{-4}$

Negative Exponents: $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

Example 2: Simplify:

A) $\frac{1}{x^{-3}}$

$$\text{B)} \quad \frac{1}{3^{-4}}$$

$$\text{C)} \quad \frac{p^{-4}}{q^{-9}}$$

$$\text{D)} \quad \frac{5^{-3}}{2^{-5}}$$

Exponent Rules:

$$1. \quad a^m a^n = a^{m+n}$$

$$2. \quad \frac{a^m}{a^n} = a^{m-n}$$

$$3. \quad (a^m)^n = a^{mn}$$

$$4. \quad (ab)^n = a^n b^n$$

$$5. \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Example 3: Simplify:

$$\text{A)} \quad \left(\frac{2}{3}\right)^{-3}$$

$$\text{B)} \quad \frac{(x^3)^4 x}{x^7}$$

$$\text{C)} \quad \left(\frac{3a^2}{b}\right)^{-3}$$

$$\text{D)} \quad \frac{4^{-1}x^{-3}y}{4^{-3}x^2y^{-6}}$$

$$\text{E)} \quad (y^{-3}z^6)^{-6}$$

F) $\left(\frac{2x^3y}{xy^{-1}}\right)^3$

Scientific Notation: written in the form $a \times 10^r$

Example 4: Write the number in scientific notation:

A) 367,000,000

B) 20,250,000,000

C) 0.0000037

D) 0.00085

Example 5: Write each number in standard form:

A) 1.02×10^5

B) 8.4×10^7

C) 7.358×10^{-3}

D) 3.007×10^{-5}

Section 5.6: Dividing Polynomials

Dividing a polynomial by a monomial: $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$

Example 1: Divide:

A) $\frac{6m^2 + 2m}{2m}$

B) $\frac{25x^3 + 5x^2}{5x^2}$

$$C) \quad \frac{16x^6 + 20x^3 - 12x}{4x^2}$$

$$D) \quad \frac{15x^4y^4 - 10xy + y}{5xy}$$

Dividing a polynomial by a polynomial: use long division

Example 2: Divide $x^2 + 7x + 12$ by $x + 3$

Example 3: Divide $6x^2 + 7x - 7$ by $2x - 1$

Example 4: Divide $4x^2 + 8x - 7$ by $2x + 1$

Example 5: Divide $x^3 - 1$ by $x - 1$

Example 6: Divide $3x^4 - 2x^3 - 3x^2 + x + 4$ by $x^2 + 2$

Section 5.7: Synthetic Division

Synthetic Division can only be used when the divisor is of the form $x + k$ or $x - k$.

Example 1: Divide: $\frac{2x^3 - x^2 - 13x + 1}{x - 3}$

Example 2: Divide: $\frac{x^4 + 3x^3 - 5x^2 + 6x + 12}{x + 3}$

Example 3: Divide: $\frac{2x^3 - 35x + 12}{x - 4}$

Remainder Theorem: If a polynomial $P(x)$ is divided by $x - k$, then the remainder is $P(k)$.

Example 4: For the polynomial $P(x) = 2x^3 - 4x^2 + 5$, find $P(2)$

Example 5: For the polynomial $P(x) = 6x^5 - 4x^4 - 40x - 10$, find $P(-1)$.

Section 6.1: The Greatest Common Factor and Factoring By Grouping

Greatest Common Factor (GCF): the largest term that multiplies to every given term

Example 1: Find the GCF:

A) 28 and 40
 Factors of 28: 1, 2, 4, 7, 14, 28
 Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40
 GCF = 4

B) 55 and 21
 Factors of 55:
 Factors of 21:
 GCF =

C) 15, 18, and 66
 Factors of 15:
 Factors of 18:
 Factors of 66:
 GCF =

D) $x^3, x^7, \text{ and } x^5$
 $\text{GCF} = x^3$

E) $y, y^4, \text{ and } y^8$
 $\text{GCF} =$

F) $6x^2, 10x^3, \text{ and } 8x$
 $\text{GCF} =$

G) $a^3b^2, a^5b, \text{ and } a^6b^3$
 $\text{GCF} =$

Example 2: Factor out the GCF:

A) $8x + 14$
 The GCF of $8x$ and 14 is 2 .
 $8x + 14 = 2(4x + 7)$
 To check, distribute the 2 .

B) $6x + 18$

C) $x^8 + x^4$

Example 3: Factor out the gcf:

A) $6a^4 - 12a$

B) $-9y^5 + 18y^2 - 3y$
 (Hint: factor out a -1)

C) $5(x + 3) + y(x + 3)$
 The GCF is $(x + 3)$
 $5(x + 3) + y(x + 3) = (x + 3)(5 + y)$

D) $7xy^3(p + q) - (p + q)$

Example 4: Factor by grouping: $xy + 2x + 3y + 6$

Group together first two terms and second 2 terms:

$$(xy + 2x) \text{ and } (3y + 6)$$

In each grouping, factor the GCF:

$$\begin{aligned} &(xy + 2x) + (3y + 6) \\ &= x(y + 2) + 3(y + 2) \end{aligned}$$

The $(y + 2)$ is the GCF, so factor it out

$$= (y + 2)(x + 3)$$

To check the answer, FOIL

$$(y + 2)(x + 3) = yx + 3y + 2x + 6, \text{ our original question}$$

Example 5: Factor by grouping: $ab + 4a + 7b + 28$
 $(ab + 4a) + (7b + 28)$

Example 6: Factor by grouping: $2x^2 + 5xy + 2x + 5y$

Example 7: Factor by grouping: $3xy + 2 - 3x - 2y$
 $3xy + 2 - 3x - 2y = 3xy - 3x + 2 - 2y$

Example 8: Factor by grouping:
 $4xy + 15 - 12x - 5y$

Example 9: Factor by grouping: $x^3 + 2x^2 + 4x - 7$

Section 6.2: Factoring Trinomials of the Form $x^2 + bx + c$

Example 1: Factor: $x^2 + 7x + 12$

To factor as 2 binomials, we need numbers that multiply to the last term, 12, and add to the middle term, 7.

<u>Multiply to 12</u>	<u>Add to 7</u>
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1 * 12	no
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2 * 6	no
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3 * 4	yes
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Only 3 and 4 fit both conditions.

$$x^2 + 7x + 12 = (x + 3)(x + 4) \text{ or } (x + 4)(x + 3)$$

Example 2: Factor: $x^2 + 5x + 6$

<u>Multiply to 6</u>	<u>Add to 5</u>
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Example 3: Factor: $x^2 - 12x + 35$

<u>Multiply to 35</u>	<u>Add to -12</u>
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Example 4: Factor: $x^2 + 4x - 12$

<u>Multiply to -12</u>	<u>Add to 4</u>
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Example 5: Factor: $y^2 - y - 42$

<u>Multiply to -42</u>	<u>Add to -1</u>
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Signs:

$$x^2 + bx + c = (x + \underline{\quad})(x + \underline{\quad}) \leftarrow \text{All signs positive}$$

$$x^2 - bx + c = (x - \underline{\quad})(x - \underline{\quad}) \leftarrow \text{First sign is negative but second sign is positive gives negatives}$$

$$x^2 + bx - c \text{ or } x^2 - bx - c = (x - \underline{\quad})(x + \underline{\quad}) \leftarrow \text{Second sign is negative gives one positive and one negative}$$

Example 6: Factor: $x^2 - 17x + 70$

Example 7: Factor: $y^2 - 2y - 36$

Example 8: Factor: $a^2 + 2a + 10$

Example 9: Factor: $x^2 + 7xy + 6y^2$

Example 10: Factor: $x^4 + 13x^2 + 12$

Example 11: Factor: $40 - 13z + z^2$

Example 12: Factor completely: $3x^2 - 24x - 60$
First, factor out the gcf. Then factor as 2 binomials.

Example 13: Factor completely: $5x^5 - 25x^4 - 30x^3$

Section 6.3: Factoring Trinomials of the Form $ax^2 + bx + c$ and Perfect Square Trinomials

Example 1: Factor using reverse FOIL: $3x^2 + 11x + 6$

We know our answer will be in the form $(\quad + \quad)(\quad + \quad)$

Since the first term of the trinomial is $3x^2$, the binomials must start with $3x$ and x , because only those terms multiply together to give $3x^2$. So we get $(3x + \quad)(x + \quad)$

As for the last terms in each binomial, their product must be 6, the last term in the trinomial. There are two ways to multiply to 6: 3 times 2 and 6 times 1. To factor with reverse FOIL, place each possibility in the binomials and find if the sum of the outer and inner terms is $11x$, the middle term in the trinomial. The one that does give $11x$ is the correct factoring.

Try 1 and 6:

$$(3x + 1)(x + 6) = 3x^2 + 18x + 6x + 6 = 3x^2 + 24x + 6 \neq 3x^2 + 11x + 6 \text{ so it doesn't work}$$

Try 2 and 3:

$$(3x + 2)(x + 3) = 3x^2 + 9x + 2x + 6 = 3x^2 + 11x + 6 \text{ so this is our answer.}$$

Example 2: Factor using reverse FOIL: $6x^2 - 5x + 1$

Example 3: Factor using reverse FOIL: $2x^2 - 11x + 12$

Example 4: Factor using reverse FOIL: $35x^2 + 4x - 4$

Example 5: Factor using reverse FOIL: $4x^2 + 12x + 5$

Example 6: Factor using reverse FOIL: $14a^2 - 3ab - 2b^2$

Example 7: Factor using reverse FOIL: $5x^4 + 34x^2 - 7$

Example 8: Factor completely: $6xy^2 + 33xy - 18x$

Example 9: Factor completely: $12x^5 + 10x^4 - 42x^3$

Perfect Square Trinomials:

1. $a^2 + 2ab + b^2 = (a + b)^2$
2. $a^2 - 2ab + b^2 = (a - b)^2$

Example 10: Factor completely: $x^2 + 20x + 100$

Example 11: Factor completely: $x^2 - 14x + 49$

Example 12: Factor completely: $9x^2 - 20xy + 4y^2$

Section 6.4: Factoring Trinomials of the Form $ax^2 + bx + c$ by Grouping

Example 1: Factor using grouping: $3x^2 + 31x + 10$

Multiply the leading coefficient and the constant term. Here, we get 30. Find numbers that multiply to 30 and add to the middle coefficient, 31. 30 and 1 should fit both conditions. Rewrite the middle term using the 30 and 1 that we just found:

$$3x^2 + 31x + 10 = 3x^2 + 30x + 1x + 10$$

Since we now have 4 terms, use factoring by grouping to finish.

$$(3x^2 + 30x) + (x + 10) = 3x(x + 10) + 1(x + 10) = (3x + 1)(x + 10) \text{ which is our answer}$$

Example 2: Factor using grouping: $6x^2 - 13x + 7$

Example 3: Factor using grouping: $6x^2 - 7x - 5$

Example 4: Factor using grouping: $2x^2 - 7x + 5$

Example 5: Factor completely: $14x^2 - 3x - 2$

Example 6: Factor completely: $12y^5 + 10y^4 - 42y^3$

Section 6.5: Factoring Binomials

Difference of 2 Squares:

$$a^2 - b^2 = (a + b)(a - b)$$

Example 1: Factor completely: $x^2 - 25$

Example 2: Factor completely: $4x^2 - 1$

Example 3: Factor completely: $x^4 - 16$

Example 4: Factor completely: $y^4 - 1$

Example 5: Factor completely: $4x^3 - 49x$

Example 6: Factor completely: $x^2 + 1$

Sum of 2 Cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Difference of 2 Cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Example 7: Factor completely: $x^3 + 8$

Example 8: Factor completely: $x^3 - 64$

Example 9: Factor completely: $27x^3 + 64y^3$

Factoring Review

How to factor completely:

Step 1: If possible, factor out a GCF.

Step 2: Look at how many terms you have.

2 terms:

If possible, factor as difference of 2 squares.

If possible, factor as sum of 2 cubes.

If possible, factor as difference of 2 cubes.

If none of those works, it can't be factored further.

3 terms:

If possible, factor as a perfect square trinomial.

If the leading coefficient is 1, you need 2 numbers that multiply to the last term and add to the middle term. If there are no such terms, it can't be factored further.

If the leading coefficient isn't 1, factor by reverse FOIL or by grouping.

4 terms:

Use factoring by grouping.

Step 3: See if any of the factored polynomials can be factored further.

Step 4: Check by multiplying.

Section 6.6: Solving Quadratic Equations by Factoring

Quadratic Equation: an equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$.

Zero Factor Theorem: if $ab = 0$, then $a = 0$ or $b = 0$.

Example 1: Solve:

$$(x - 3)(x + 1) = 0$$

Use the zero factor theorem and solve.

Example 2: Solve:

$$x(4x - 3) = 0$$

Example 3: Solve:

$$x^2 - 3x - 18 = 0$$

Step 1: Get a zero on one side. Here, we already have a zero.

Step 2: Factor.

$$x^2 - 3x - 18 = 0$$

$$(x - 6)(x + 3) = 0$$

Step 3: Use the zero factor theorem and solve.

$$x - 6 = 0 \text{ or } x + 3 = 0$$

$$x = 6 \text{ or } -3 \quad \text{When solving quadratics, we should get 2 answers.}$$

Example 4: Solve:

$$25x^2 - 29x = -4$$

Example 5: Solve:

$$x(x - 4) = 5$$

Example 6: Solve:

$$x^2 + 6x + 9 = 0$$

Example 7: Solve:

$$-5x^2 + 20x + 60 = 0$$

Example 8: Solve:

$$2x^3 - 18x = 0$$

Example 9: Solve:

$$3x^3 - 9x^2 - 12x = 0$$

Section 6.7: Quadratic Equations and Problem Solving

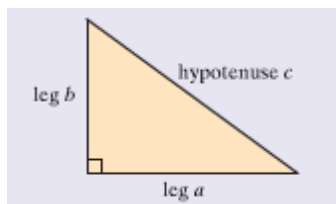
Example 1: An object is dropped from the roof of a 144-foot building. Neglecting air resistance, the height h in feet of the object above ground after t seconds is given by the equation $h = -16t^2 + 144$. How long will it take for the object to hit the ground?

Example 2: The square of a number plus three times the number is 70. Find the number.

Example 3: The height of a triangular sail is 2 meters less than twice the length of the base. If the sail has an area of 30 square meters, find the base and the height.

Example 4: Find 2 consecutive integers whose product is 34 more than their sum.

Pythagorean Theorem: In a right triangle, the sum of the squares of the legs equals the square of the hypotenuse.



$$a^2 + b^2 = c^2$$

Example 5: Find the lengths of the sides of a right triangle if the lengths can be represented as three consecutive even integers.

Section 7.1: Rational Functions

Rational Function: of the form $f(x) = \frac{p(x)}{q(x)}$ where p and q are polynomials and $q(x) \neq 0$.

Example 1: Find the domain of the rational function.

A) $f(x) = \frac{5x^2 - 3}{x - 1}$

For the domain, the only thing we need to worry about is that we don't divide by zero. So set the denominator not equal to zero and solve.

$$\text{B)} \quad f(x) = \frac{7x-2}{x^2-2x-15}$$

$$\text{C)} \quad f(x) = \frac{8x-3}{x^2-5x+6}$$

Example 2: Write the fraction in lowest terms / reduce:

$$\text{A)} \quad \frac{7}{14}$$

$$\text{B)} \quad \frac{2x^2}{10x^3-2x^2}$$

Step 1: factor.

$$\frac{2x^2}{10x^3-2x^2} = \frac{2x^2}{2x^2(5x-1)}$$

Step 2: cross out any terms that are in both the numerator and denominator.

$$= \frac{1}{5x-1}$$

$$\text{C)} \quad \frac{x+5}{x^2-4x-45}$$

$$\text{D)} \quad \frac{9x^2+13x+4}{8x^2+x-7}$$

$$\text{E)} \quad \frac{x+2}{2+x}$$

$$\text{F)} \quad \frac{3-x}{x-3}$$

$$G) \quad \frac{18-2x^2}{x^2-2x-3}$$

$$H) \quad \frac{x^3+8}{x+2}$$

Section 7.2: Multiplying and Dividing Rational Expressions

Example 1: Multiply the fractions and write your answer in lowest terms:

$$A) \quad \frac{25x}{2} \cdot \frac{1}{y^3}$$

To multiply fractions, we multiply the numerators and denominators then reduce.

$$\frac{25x}{2} \cdot \frac{1}{y^3} = \frac{25x}{2y^3}$$

$$B) \quad \frac{-7x^2}{5y} \cdot \frac{3y^5}{14x^2}$$

$$C) \quad \frac{x^2-x}{5x} \cdot \frac{15}{x^2-1}$$

$$D) \quad \frac{3x+3}{5x-5x^2} \cdot \frac{2x^2+x-3}{4x^2-9}$$

$$E) \quad \frac{6-3x}{6x+6x^2} \cdot \frac{3x^2-2x-5}{x^2-4}$$

$$F) \quad \frac{5x-20}{3x^2+x} \cdot \frac{3x^2+13x+4}{x^2-16}$$

Example 2: Divide the fractions and write your answer in lowest terms:

$$A) \quad \frac{3x^3y^7}{40} \div \frac{4x^3}{y^2}$$

Division by a fraction is the same thing as multiplying by the reciprocal.

$$\frac{3x^3y^7}{40} \div \frac{4x^3}{y^2} = \frac{3x^3y^7}{40} \cdot \frac{y^2}{4x^3}$$

$$B) \quad \frac{x^2+x-2}{10} \div \frac{2x+4}{5}$$

$$C) \quad \frac{6x+2}{x^2-1} \div \frac{3x^2+x}{x-1}$$

$$D) \quad \frac{2x^2-11x+5}{5x-25} \div \frac{4x-2}{10}$$

$$E) \quad \frac{35x-7x^2}{x^2-25} \div \frac{x^2+4x}{x^2+3x-10}$$

Section 7.3: Adding and Subtracting Rational Expressions I

Example 1: Add or subtract the fractions and write your answer in lowest terms:

$$A) \quad \frac{6}{5} + \frac{2}{5}$$

To add or subtract fractions with like denominators, add or subtract numerators. Then reduce (you may first have to factor before reducing).

$$B) \quad \frac{5m}{2n} + \frac{m}{2n}$$

$$C) \quad \frac{2y}{2y-7} - \frac{7}{2y-7}$$

$$D) \quad \frac{3x^2+2x}{x-1} - \frac{10x-5}{x-1}$$

When subtracting a fraction that has 2 or more terms in the numerator, make sure to distribute the negative sign to *all* terms in the second numerator.

$$E) \quad \frac{4x^2+15x}{x+3} - \frac{8x+15}{x+3}$$

Least Common Divisor: the lcd is the smallest number that can be evenly divided by all the given terms.

Example 2: Find the lcd:

$$A) \quad 8, 22$$

$$B) \quad 5x, 15x^2$$

$$C) \quad x + 2, x - 2$$

$$D) \quad x, x + 4$$

$$E) \quad 3x + 15, (x + 5)^2$$

$$F) \quad x^2 - x - 6, x^2 + 3x + 2$$

Section 7.4: Adding and Subtracting Rational Expressions II

Example 1: Add or subtract and write your answer in lowest terms:

A) $\frac{4}{3} + \frac{1}{2}$

B) $\frac{3}{10x^2} + \frac{7}{25x}$

Step 1: Find the lcd (you may have to factor the denominators first).

$$\text{lcd} = 50x^2$$

Step 2: Rewrite so that each fraction has the lcd. You may have to change the numerators as well.

$$\frac{3}{10x^2} + \frac{7}{25x} = \frac{3 \cdot 5}{50x^2} + \frac{7 \cdot 2x}{50x^2} = \frac{15}{50x^2} + \frac{14x}{50x^2}$$

Step 3: Add or subtract the numerators so that you have a single fraction. Remember to subtract the *entire* second numerator.

$$\frac{15}{50x^2} + \frac{14x}{50x^2} = \frac{15+14x}{50x^2}$$

Step 4: Reduce (you may have to factor the numerator first). Here, the numerator does not factor, so the fraction cannot be reduced.

C) $\frac{6x}{x^2-4} - \frac{3}{x+2}$

D) $\frac{12x}{x^2-25} - \frac{6}{x+5}$

E) $\frac{2}{3x} + \frac{5}{x+1}$

$$\text{F)} \quad \frac{7}{x-3} - \frac{9}{3-x}$$

Here, the denominators are opposite. Multiply the second fraction by $-1/-1$ to get the same denominator.

$$\text{G)} \quad \frac{6}{x-5} + \frac{7}{5-x}$$

$$\text{H)} \quad \frac{3}{2x^2+x} - \frac{2x}{6x+3}$$

$$\text{I)} \quad \frac{5}{2x^2+3x} - \frac{3x}{4x+6}$$

$$\text{J)} \quad \frac{2x}{x^2+2x+1} + \frac{x}{x^2-1}$$

$$\text{K)} \quad \frac{2x}{x^2+7x+12} + \frac{3x}{x^2-9}$$

Section 7.5: Solving Equations with Rational Expressions

Example 1: Solve:

$$\frac{x}{2} + \frac{8}{3} = \frac{1}{6}$$

Step 1: Multiply both sides by the lcd and distribute. This means you will be multiplying every numerator on both sides by the lcd. (The denominators do not change).

$$\frac{6}{1} \left(\frac{x}{2} + \frac{8}{3} \right) = \frac{6}{1} \left(\frac{1}{6} \right)$$

$$\frac{6x}{2} + \frac{48}{3} = \frac{6}{6}$$

Step 2: Reduce the fractions. If you did step 1 correctly, every denominator should cross out.

$$3x + 16 = 1$$

Step 3: Solve.

$$3x = -15$$

$$x = -5$$

Step 4: Check to make sure that your answer(s) do not cause division by zero in the original problem. Here, there are no x's in the denominators, so our answer is fine.

Example 2: Solve:

$$\frac{x-4}{2} - \frac{x-3}{9} = \frac{5}{18}$$

Example 3: Solve:

$$3 - \frac{6}{x} = x + 8$$

Example 4: Solve:

$$2 + \frac{6}{x} = x + 7$$

Example 5: Solve:

$$\frac{4x}{x^2+x-30} + \frac{2}{x-5} = \frac{1}{x+6}$$

Example 6: Solve:

$$\frac{7x}{x^2+2x-15} - \frac{4}{x-3} = \frac{-2}{x+5}$$

Example 7: Solve:

$$\frac{2x}{x-4} = \frac{8}{x-4} + 1$$

Example 8: Solve:

$$\frac{5x}{x-1} = \frac{5}{x-1} + 3$$

Example 9: Solve:

$$x + \frac{14}{x-2} = \frac{7x}{x-2} + 1$$

Example 10: Solve:

$$x - \frac{6}{x+3} = \frac{2x}{x+3} + 2$$

Example 11: Solve the equation for x:

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{x}$$

Section 7.6: Applications of Rational Equations

Example 1: Solve:

$$\frac{45}{x} = \frac{5}{7}$$

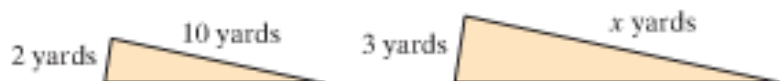
To solve an equation with one fraction on each side, cross multiply to get the numerator of the first times the denominator of the second equal to the denominator of the first times the numerator of the second.

Example 2: Solve:

$$\frac{x-5}{3} = \frac{x+2}{5}$$

Example 3: 3 boxes of CDs costs \$37.47. How much will 5 boxes of CDs cost?

Example 4: Find x given the similar triangles:



Example 5: The quotient of a number and 6, minus $\frac{5}{3}$, is the quotient of the number and 2. Find the number.

Rate of Work: If a job takes x time, the rate of work is $\frac{1}{x}$

Example 6: Sally takes 3 hours to paint a room. Frank takes 7 hours to paint the same room. How long will it take if they work together?

Example 7: A car travels 180 miles in the same time it takes a truck to go 120 miles. If the car's speed is 20 mph faster than the truck's speed, find the two speeds.

Use the formula $d = rt$ to solve this.

Section 7.7: Simplifying Complex Fractions

Example 1: Simplify:

$$\frac{\frac{2x}{27y^2}}{\frac{6x^2}{9}}$$

Example 2A: Simplify with method 1:

$$\frac{\frac{x}{y^2} + \frac{1}{y}}{\frac{y}{x^2} + \frac{1}{x}}$$

Step 1 for method 1: Add or subtract the fractions in the numerator and the denominator.

$$\frac{\frac{x}{y^2} + \frac{1}{y}}{\frac{y}{x^2} + \frac{1}{x}} = \frac{\frac{x}{y^2} + \frac{y}{y^2}}{\frac{y}{x^2} + \frac{x}{x^2}} = \frac{\frac{x+y}{y^2}}{\frac{y+x}{x^2}}$$

Step 2 for method 1: Rewrite as multiplication and simplify.

$$\frac{\frac{x+y}{y^2}}{\frac{y+x}{x^2}} = \frac{x+y}{y^2} \cdot \frac{x^2}{y+x} = \frac{x^2}{y^2}$$

Example 2B: Simplify with method 2:

Step 1 for method 2: Multiply the fraction by the lcd of all the fractions.

$$\left(\frac{\frac{x}{y^2} + \frac{1}{y}}{\frac{y}{x^2} + \frac{1}{x}} \right) \left(\frac{\frac{x^2 y^2}{1}}{\frac{1}{x^2 y^2}} \right) = \frac{\frac{x^3 y^2}{y^2} + \frac{x^2 y^2}{y}}{\frac{x^2 y^3}{x^2} + \frac{x^2 y^2}{x}}$$

Step 2 for method 2: Reduce. You may have to do this in more than one part.

$$\frac{\frac{x^3 y^2}{y^2} + \frac{x^2 y^2}{y}}{\frac{x^2 y^3}{x^2} + \frac{x^2 y^2}{x}} = \frac{x^3 + x^2 y}{y^3 + x y^2} = \frac{x^2(x+y)}{y^2(y+x)} = \frac{x^2}{y^2}$$

Example 3: Simplify:

$$\frac{\frac{2}{x+1} + \frac{3}{x}}{\frac{4}{x} - \frac{1}{x+1}}$$

Example 4: Simplify:

$$\frac{\frac{3}{x-4} - \frac{2}{4-x}}{\frac{2}{x-4} - \frac{2}{x}}$$